Assessment of Safety Factors of natural slopes based on the Limit Equilibrium Method and Artificial Neural Networks

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RESUMO: A compreensão da estabilidade dos taludes é fundamental para uma gestão eficaz dos riscos e para a prevenção de catástrofes naturais. Acidentes relacionados com o deslizamento de encostas naturais podem afetar seriamente sociedades e sua infraestrutura, com consequências graves como a perda de vidas humanas e perdas econômicas. Face às incertezas associadas às características das camadas de solo e às alterações climáticas, os modelos de redes neuronais artificiais (RNA) surgem como soluções rápidas e robustas para a previsão do fator de segurança de taludes naturais. Nesse sentido, neste trabalho compilou-se um extenso conjunto de dados, relacionando parâmetros de resistência do solo (coesão e ângulo de atrito) e fatores geométricos do talude (inclinação, espessura da camada de solo e a espessura de solo saturado) com fatores de segurança determinados através de um método de equilíbrio limite. Uma porcentagem de 80% dos dados foi utilizada para treinar e otimizar os hiperparâmetros do modelo de RNA empregando uma abordagem de otimização Bayesiana. O restante dos modelos foi utilizado para a execução de testes e validação. Adicionalmente, foi efetuado um estudo da importância das características para investigar o efeito de cada parâmetro na determinação do fator de segurança.

PALAVRAS-CHAVE: Estabilidade de taludes, Fator de segurança, Redes Neurais Artificiais

ABSTRACT: Understanding the slope stability is vital for effective risk management and prevention of natural disasters. These accidents can lead to extensive destruction, affecting societies, infrastructure, and agriculture, with severe consequences. In the face of uncertainties associated with soil characteristics and climate changes, artificial neural network (ANN) models raise as fast and robust solutions for predicting the safety factor (SF) of natural slopes. To train an ANN, an extensive synthetic dataset was compiled, encompassing soil strength parameters (cohesion and friction angle) and slope geometric factors (inclination and soil layer height). Next, the dataset was separated into 80% to train and optimize the model's hyperparameters using a Bayesian optimization approach. The rest of the data was used for testing and validation. Additionally, a features importance study was made to investigate the effect of each input in the SF determination.

KEYWORDS: Slope's Stability, Limit State Method, Artificial Neural Networks

1 INTRODUCTION

The stability of natural slopes holds paramount significance within geotechnical engineering. This discipline bears the responsibility of addressing and mitigating the potentially devastating risks associated with landslides which are one of the major natural disasters that produce intensive, widespread damage to human societies resulting in grave economic, social, and environmental losses to many countries (Crozier, 1999, 2010; Qi and Tang, 2018).

Safety factors (SF) are usually adopted in order to check if a slope is stable or not. In theory, when the safety factor is higher than one, the slope is stable. However, due to parameter uncertainty and security issues, geotechnical engineers consider safety factors higher than a minimum valor can vary from 1.2 – 1.5, depending on field application. Different limit equilibrium methods based on the quantification of the driving and the resisting forces along a failure surface (Li et al., 2016) are used for a quick assessment of safety factors (Bishop, 1955; Janbu et al., 1956; Spencer, 1967; Morgenstern and Price, 1965). However, a set of shear-strength parameters (cohesion and friction angle) as well as a failure or slip surface inside the slope must be assumed before computing safety factors. Such issues have been discussed in many studies with special attention and careful investigation (Michalowski, 1995; Qi and Tang, 2018; Qian et al., 2019). In this sense, a comprehensive understanding of geometrical and physical parameters is imperative in order to evaluate their impacts on slope stability problems (Wang, Xu and Xu, 2005). Several sensitivity studies were used for such purposes, combining deterministic analyses (LEM, analytical and numerical methods) and statistical methods (Sloan, 1988; Kardani et al., 2021).

Recently, computational intelligence models have gained notorious relevance in applications for several engineering problems, including soil and rock mechanics (Kardani et al., 2021). Methodologies, such as those based on support vector machines (Chang and Chiang, 2009), decision trees (Glastonbury and Fell, 2008), ensembles (Qi and Tang, 2018; Kardani et al., 2021) and artificial neural networks (Wang, Xu and Xu, 2005; Li et al., 2016; Qian et al., 2019) emerge as an important alternative to enable the development of fast, robust, and accelerated models that learn patterns and correlations between the parameters related to slopes stability and the safety factor value. Consequently, artificial intelligence-based approaches have also been used for the assessment of slope stability factors (Brand, Premchitt and Phillipson, 1984; Sloan, 1988; Crozier, 1999; Dai and Lee, 2001; Cai and Ugai, 2004; Collins and Znidarcic, 2004; Wang, Xu and Xu, 2005; Chang and Chiang, 2009; Li et al., 2016; Qi and Tang, 2018; Kardani et al., 2021).

In the present study, an artificial neural network based on a Bayesian optimization approach is proposed for the assessment of safety factors in natural slopes considering representative geometries and soil strength parameters. In addition, the impact of rain infiltrations affecting particularly the weight of the soil are included through the consideration of a shallower and saturated soil thickness.

2 METHODOLOGY

The accuracy of neural networks is mainly dependent on a set of variables, such as the quality and size of the dataset, the structure of the model, and the number of training epochs. In this sense, the dataset generation procedures and the optimization scheme for obtaining a high-performance artificial neural network model are described in the following sections.

2.1 Artificial Neural Networks

During the last decades, the use of ANN has presented a crescent use nowadays in several applications. Deep learning techniques raise as great alternatives due to the flexibility and robustness of neural networks. In this context, ANN was developed inspired by the structure of a human brain composed of layers and neurons analogous to synaptic connections, where each performs operations indicating the level of activation of each input in each neuron (HAYKIN, 2008), as expressed in the equation (1).

$$\boldsymbol{y}^L = \sigma^L(\boldsymbol{W}^L\,\boldsymbol{y}^{L-1} + \boldsymbol{B}^L) \tag{1}$$

where $\boldsymbol{W}^L, \boldsymbol{B}^{\boldsymbol{L}}, \boldsymbol{y}^L$ and σ^L are the weights matrix, bias vector, outputs, and activation of the layer $\boldsymbol{L}$, respectively. It is important to mention that for $\boldsymbol{L}$ equal to 1, $\boldsymbol{y}^1$ is the input vector and for $\boldsymbol{L}$ equal to the number of ANN layers, $\boldsymbol{y}^L$ is the output of the model.

The activation function is introduced to regard the nonlinear capacity of the model for learning complex function relationships between inputs and outputs. The most straightforward adopted activation functions are rectified linear unit (ReLU), sigmoid, hyperbolic tangent, etc. Recently, a new class of adaptative loss functions has been proposed showing a new class of activations capable of enhancing the accuracy of ANN (BARRON, 2019).

Figure 1 shows the fully connected neural network, the so-called dense feedforward neural network, is a set of layers composed of artificial neurons that map the inputs $\boldsymbol{x_i}$ to the outputs $\boldsymbol{y_i}$, after passing the information by the layers in a forward pass from the input to the output layer. The internal layers, i.e., the layers between the input and output layers, are called hidden layers and define the deepness of the model.

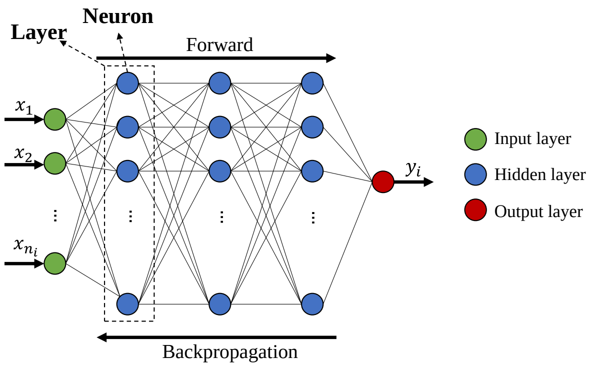

Figure 1. Fully connected neural network representation.

The ANN training strategy, when performed in a supervised manner, consists of minimizing an objective function, denoted as a loss function ($\mathcal{L}$), seeking to determine the optimal weights and biases ($W^*; b^*$) of the model (GOODFELLOW; BENGIO; COURVILLE, 2016; HAYKIN, 2008). Typically, error metrics are used as loss functions, which consider the difference between predicted (y_i^{pred}) and expected (y_i^{real}) responses, as shown in equation (2) represented by the mean squared error (MSE).

$$\mathcal{L}(W^*, b^*) = \arg\min\left[\frac{1}{N}\sum_{i=1}^{N}\left|y_i^{real} - y_i^{pred}\right|^2\right] \tag{2}$$

where N denotes the total number of labeled data.

The optimization of the ANN parameters is a numerical process that uses the gradient of the loss function concerning the weights and biases in an interactive way. Then, after a forward pass, the gradient of loss is obtained using automatic differentiation (GÜNEȘ BAYDIN et al., 2018), and the parameters are adjusted often using gradient-based schemes in the backpropagation pass by applying the equations (3) and (4).

$$W^{k+1} = W^k - \eta\frac{\partial\mathcal{L}}{\partial W^k} \tag{3}$$

$$B^{k+1} = B^k - \eta\frac{\partial\mathcal{L}}{\partial B^k} \tag{4}$$

2.2 Limit equilibrium methods for safety factor determination

The mathematical formulation of LEM involves the equilibrium between the driving and resisting forces along a potential failure surface within the slope. The analysis considers the forces acting parallel and perpendicular to the assumed failure surface. The basic principle is to determine the critical conditions under which the resisting forces along a potential failure surface are just sufficient to balance the driving forces. LEM often rely on some assumptions, such as the existence of homogeneous soil properties and a single and pre-defined failure surface, making those methods suitable for quick assessments of safety factors. However, LEM require careful consideration of their limitations in more complex geological settings.

In this study, the Morgenstern and Price method was employed for the assessment of safety factors (MORGENSTERN; PRICE, 1965). This method extends the original Bishop method (BISHOP, 1955) to consider circular or non-circular failure surfaces adopting a series of slices to analyze the mobilization of shear strength. While more complex than the original Bishop method, the Morgenstern and Price method provides a more refined analysis for slope stability, particularly in situations where the failure surface shape is not strictly circular (HUANG, 2014a). In this study, the safety factor is rigorously assessed through the application of limit equilibrium analyses using an open software named HYRCAN® (MIKOLA, R, 2023).

3 PROBLEM SETUP

3.1 Dataset Generation

Several geometries using different soil layer thickness (h) and slope inclination (α) as well as different soil strength parameters given by the soil cohesion (c) and friction angle (ϕ) were used as design variables in order to represent a wide range of soils and slopes. In addition, a saturated soil layer is included in order to represent effects of rain infiltration which can increase the weight of the soil. Therefore, two soil layers with different soil unit weights are considered. The shallower layer, represented by the height (h_w), is the saturated soil with a unit weight (γ_{sat}) of 21 kN/m^3. The deeper layer, represented by height ($h - h_w$), is the soil in natural conditions with a unit weight (γ_{nat}) of 18 kN/m^3. The parametric model is shown in Figure 2.

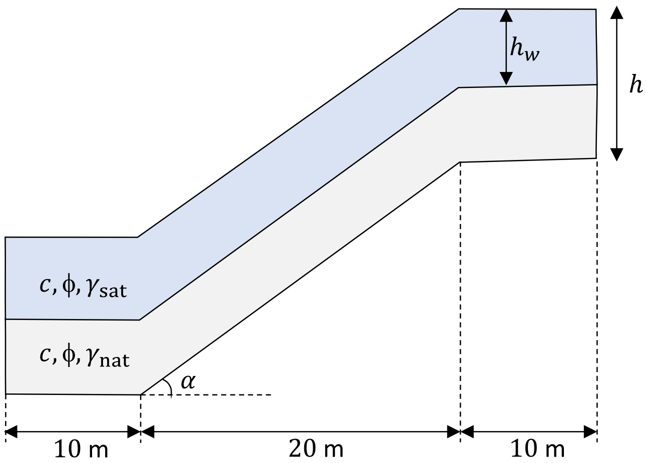

Figure 2. Graphical representation of the parametric slope.

Some limits reported in the literature were adopted to generate the matrix of experiments as shown in Table 1. In total, 10,500 numerical models compound the extensive dataset that ensures a robust foundation for subsequent analyses under diverse and systematically varied conditions. Such a meticulous sampling strategy not only contributes to the comprehensiveness of the dataset but also provides a nuanced understanding of the interplay between the input variables in the experimental setup.

Table 1. Upper and lower bounds utilized to generate the datasets.

Parameters	Symbol	Lower bound	Upper bound	Unit
Height of soil on slope	h	1.0	5.0	m
Height of saturated soil	h_W	0.1	4.5	m
Slope inclination	α	15	45	degree
cohesion	c	0	20	kPa
friction angle	ϕ	15	45	degree

In each experiment, a systematic sampling strategy was employed through the utilization of a uniform grid scheme. This approach requires discretizing all input variables, including height, inclination, cohesion, friction angle, and saturation front, within regular grids. Specifically, the height variable underwent discretization into five incremental values, while the inclination, cohesion, and friction angle were discretized with a uniform interval of 5 units each. Subsequently, the discretization process for the saturation front entailed considering six percentage of the height: 10%, 20%, 40%, 60%, 80% and 90%. The resulting discretized grids for these parameters were then individually combined, resulting in a comprehensive experiment matrix.

Considering a briefly descriptive analysis of the obtained SF values, the computed statistics reveal that the mean of Safety Factor (SF) values is 1.76, accompanied by a standard deviation of 1.21. Notably, the contrast between the relatively lower median (1.42) and the correspondingly higher upper bound (6.57) underscores the asymmetry in SF distribution, as shown in Figure 3(a). This observation implies an inherent imbalance in the distribution of SF values, a noteworthy aspect that warrants further consideration in the analysis of the simulated results.

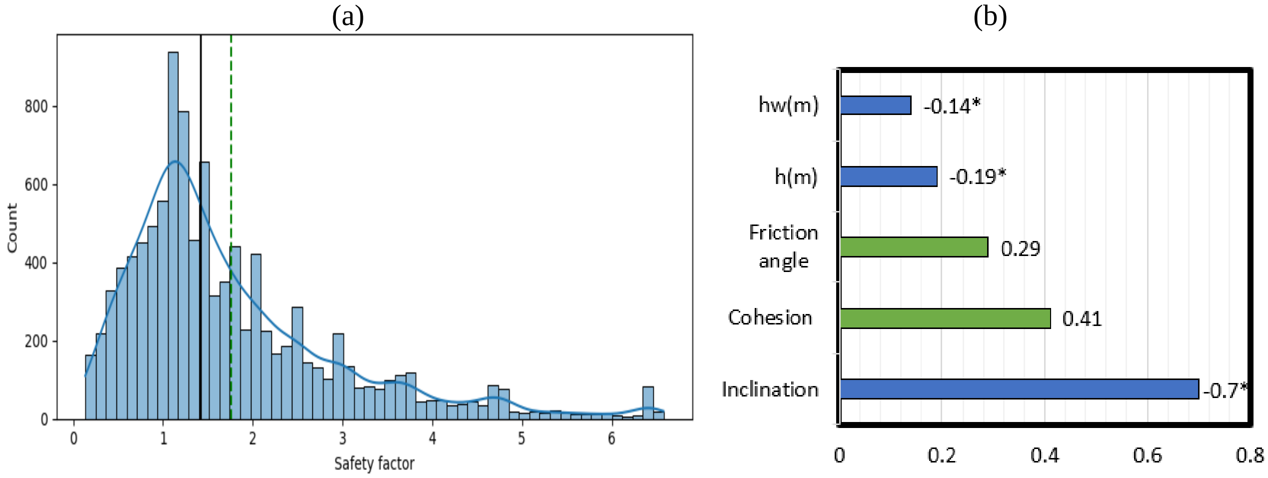

Figure 3. Exploratory analysis of synthetic dataset. (a) Histogram of safety factor values in the dataset; (b) Correlation values between input features and safety factor.

Considering the built data set, the correlation values between all features and SF are shown in Figure 3(b). Notably, the inclination of the slope reports a pronounced negative correlation value of -0.7, categorizing it as a highly correlated feature with an inverse relationship with the SF. This means that the correlation coefficient is either higher than 0.5 or lower than - 0.5. Moreover, the height and saturation front values also report a negative correlation with the safety factor, albeit at a lower magnitude. In contrast, the strength parameters (cohesion and friction angle), demonstrate positive correlation values, showing a direct correlation with SF indicating the increasing of SF with increasing of strength, which agrees with the physical behavior of slopes.

3.2 K-Fold Cross-Validation (CV)

In the evaluation of an Artificial Neural Network (ANN) model, two prevalent challenges are systematically addressed: **(i)** assessing training accuracy concerning both training and testing data, and **(ii)** evaluating predictive performance on entirely new data. The initial consideration involves gauging the risk of underfitting or overfitting by scrutinizing the metrics derived from the model's performance on both training and validation datasets. This comparative analysis serves as a crucial diagnostic to discern the model's ability to capture patterns in the data without compromising its ability to generalize to new, unseen instances. Subsequently, the model's generalization performance is further examined by assessing its predictions on novel unseen data.

To address the performance issues associated with ANNs, the cross-validation scheme in accordance to optimization algorithms are commonly used to define the optimal hyperparameters of the model, ensuring that the best model will continue to perform accurately under new possible scenarios without losing significant capacity.

The efficacy of the trained model is assessed through four conventional metrics quantifying the disparity between the predicted values (FS^*) and the corresponding expected safety factor values (FS). The metrics used here are: (i) the Mean Absolute Percentage Error (MAPE); (ii) the Mean Absolute Error (MAE); (iii) the Mean Squared Error (MSE); and (iv) the Coefficient of Determination (R^2).

3.3 Hyperparameter Tuning and Features Importance

The performance of the ANNs is very dependent on components called hyperparameters such as: the number of hidden layers, the number of neurons per layer, the activation functions, the regularization terms and many others. In the literature, the set for each of hyperparameter is defined in a previous stage to the training of the ANN considering some relevant search schemes (e.g. grid- and randomized-based search) (BERGSTRA et al., 2011; CHEN; WU; LIU, 2021). However, instead of using exhaustive search schemes, the Bayesian optimization method raises as a robust approach to optimize objective functions that require high computational effort, working well in noisy functions – as can be the case of the neural network hyperparameter tuning (ABREU et al., 2023; FRAZIER, 2018). The process of Bayesian optimization involves creating a probabilistic model with the objective of minimizing a defined performance metric, specifically the k-fold cross-validation performance represented by the average Mean Squared Error (MSE). The Latin-Hypercube-Sample (LHS) strategy is used to generate a set of initial evaluation points. Then, until the pre-defined stopping criterion is met, the probabilistic model is continuously updated as new points are computed by the optimization algorithm. This work adopts the limited-memory BFGS optimization algorithm. Moreover, the Scikit-Optimize library® (PEDREGOSA et al., 2011) is adopted to perform the Bayesian optimization.

4 RESULTS

The definition of best set of hyperparameters was taken considering 100 evaluations in the Bayesian optimization. The minimum solution is reached after 60 calls, stabilizing the objective function during the optimization process. The optimal hyperparameters for accurately building an ANN-based model for predicting SF were found to be 0.00018 for the learning rate, 4 for the number of hidden layers, 96 for the number of neurons, and rectified linear units for the activation function. The obtained values, when compared against the defined limits, show that the adopted range is adequate, as the values are not at the bounds.

To interpret the importance of each hyperparameter in the proposed ANN, the partial dependence of each one is presented in Figure 4. The results show the relevant importance of choosing a correct learning rate value and the number of hidden layers. On the other hand, the number of neurons per layer and the activation function do not report an effective impact when the values change.

Finally, Table 2 that shows low values of MAE, MSE and MAPE for the optimum hyperparameters. Consequently, it is expected a high accuracy of the best model for both training and test datasets. Moreover, the high value of R^2 indicates that the model captures the tendency of real values, which is also graphically expressed by the regression plot in Figure 5(a).

The good metrics of the model provide global insights about all prediction ranges. However, critical slopes are found in the range of safety factors up to two. Therefore, a local analysis was carried out in order to elucidate the model's local performance for the critical region. Figure 5(b) shows the histogram of MAPE for the tested dataset. The results indicate that only 0.571% and 2.19% of all test samples report relative errors higher than 10%, and 5%, respectively, demonstrating the high performance of the model.

With the test suite filtering of SF < 2.0, the average distance between predictions and expected values is approximately -0.0024 with a deviation of 0.0169. That indicates that the model is underpredicting the safety factor, favoring safety. Furthermore, the maximum error is 0.055 which also favors safety and can be used as a factor to reduce predictions, giving the model more safety. Furthermore, the low error value also indicates that the model is not overly conservative.

Table 2. Prediction metrics for the neural network model

Dataset	**MAE**	**MSE**	**MAPE (%)**	**R^2**
Training	0.01522	0.00049	1.13479	0.99965
Test	0.01536	0.00054	1.09606	0.99964

After training the predictive model, it is possible to evaluate the importance of all input variables in the definition of SF. A feature importance algorithm based on a corrupted dataset strategy denoted as Permutation Importance (PEDREGOSA et al., 2011) is used to explore the impact of each geomechanical variable on determining SF. The measures of feature importance are shown in Figure 6. The results revealed that the slope's inclination is the most impacting variable, followed by soil's strength parameters, while the last two geometric properties: the total and the saturated soil height, present the least important variables.

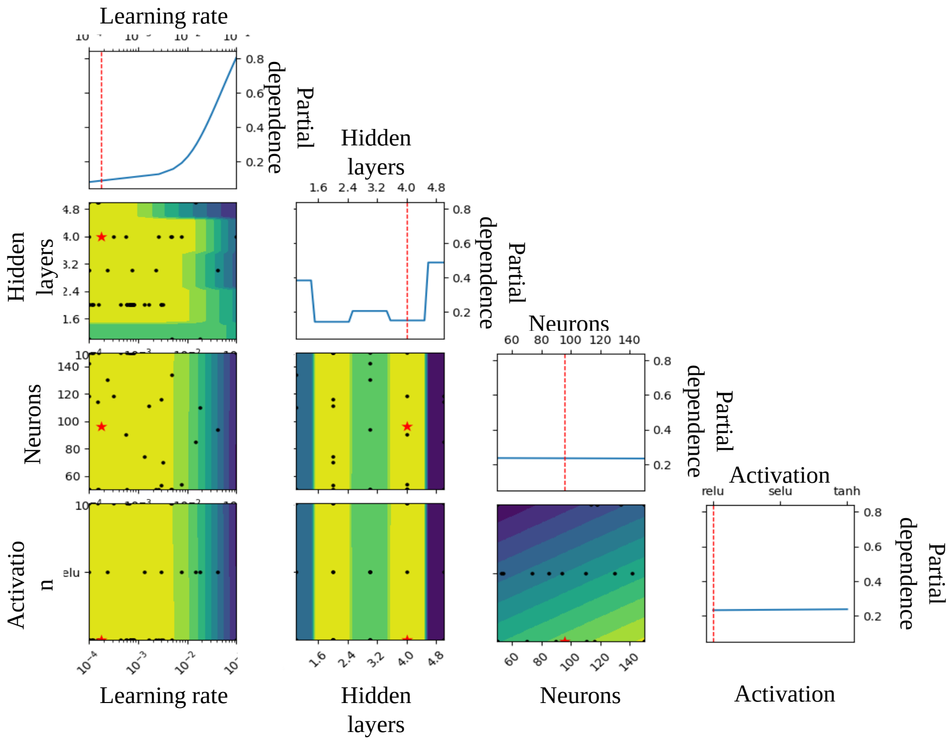

Figure 4. Partial dependence and contour plots for the Bayesian optimization results. The black dots represent the samples and the red stars, their minimum values.

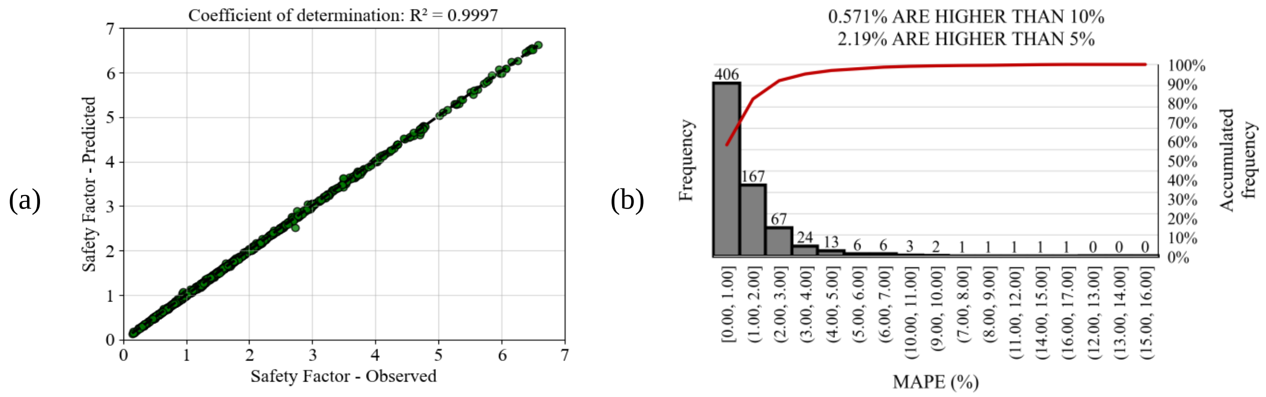

Figure 5. Artificial neural network model evaluation. (a) Regression plot (b) Histogram of errors and accumulate errors.

5. CONCLUSIONS

This study proposes an artificial intelligence-based approach for computing the safety factor of slopes considering a set of design variables related to soil strength parameters (cohesion and friction angle) and slope geometric factors (natural soil layer height, saturated soil layer height and slope inclination).

The neural network architecture was defined using a Bayesian optimization approach, considering the average performance of a 5-fold CV. After tuning, the best set of hyperparameters was chosen to train the final predictive model. Considering the test dataset, the predicted and expected responses exhibited a high degree of correlation, i.e. R^2 equals 0.9996, which indicates that the model captures quite well the SF tendency.

Moreover, the mean absolute error was 0.015, and the mean absolute relative error was 1.096%, which indicates the notorious accuracy of the proposed model. Finally, a variable importance study confirms that the inclination of slopes and cohesion are the most influential variables in assessing the stability of the slope.

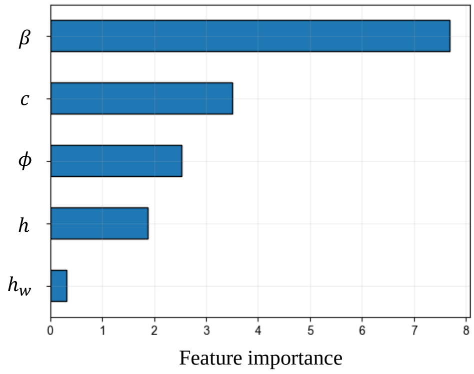

Figure 6. Variable importance for the neural network inputs.

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